Universität Hamburg
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CHAMPP
CENTER IN HAMBURG FOR
ASTRO-, MATHEMATICAL AND
PARTICLE PHYSICS

LECTURE COURSE IN THE QUANTUM UNIVERSE RESEARCH SCHOOL

**Winter Term 2024/2025**

# Ricci Flow - an introduction

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**Course Description:**

Right from its introduction by Hamilton in 1982, the Ricci flow has found applications in both geometry and topology. Perhaps the crowning achievement of the Ricci flow is the proof of the Poincaré conjecture or more generally the proof of the Thurston's Geometrization conjecture by Perelman. This course intends to be an introduction to the Ricci flow and to study many of its properties and applications. A detailed (preliminary) discussion of topics is outlined below. If the response will be good, then there could also be a "Part 2" of the course which probably will cover those results of Perelman which won't be covered in the first part.

**Topics to be covered** (the topics marked with * might be covered if there is interest among the participants.)

(1) Basics of Riemannian geometry and Ricci calculus with emphasis on calculations in local coordinates.
(2) Basics on Partial Differential Equations with a focus on parabolic PDEs; existence of solutions to such PDEs.
(3) Introduction to the Ricci flow.
(4) Short time existence using the DeTurck's trick.
(5) Evolution equations of intrinsic geometric quantities along the flow.
(6) Uhlenbeck's trick: evolution of the Riemann curvature tensor; Hamilton's theorem on positivity of Riemann curvature being preserved.
(7) Curvature estimates and long time existence.
(8) Vector bundle maximum principles; curvature pinching estimates and Hamilton–Ivey pinching estimate.
(9) Ricci flow in two dimensions. (Hamilton and Ivey's results on all compact Ricci solitons being gradient*)
(10) Li–Yau Harnack inequality and Hamilton's Harnack estimates for the Ricci flow. (Chow–Chu's approach to Hamilton's Harnack estimates using the space-time approach*)
(11) Ricci solitons item Ricci flow as a gradient flow: Perelman's $\mathcal{F}$ and $\mathcal{W}$ functionals and their monotonicity.

(12) Perelman's No Local Collapsing theorem (proof of Hamilton's little loop conjecture.)
(13) Logarithmic Sobolev inequalities.
(14) Overall idea of Perelman's proof of the Thurston's Geometrization Conjecture.

The aforementioned topics are much more than what we'll actually be able to cover in the course.

**Prerequisites:**

The target audience is Bachelors and Masters's students and PhD students so only basic knowledge of Riemannian geometry and analysis (especially PDEs) will be very beneficial.

**Literature:**

There are excellent introductions to the subject of the Ricci flow and the materials presented in the class will be followed from the references mentioned below. In particular, [2], [1] and [10] are good sources for self-study as well.

## References


[1] Bennett Chow, Peng Lu, and Lei Ni, Hamilton's Ricci flow, Graduate Studies in Mathematics, vol. 77, American Mathematical Society, Providence, RI; Science Press Beijing, New York, 2006.
[2] Bennett Chow and Dan Knopf, The Ricci flow: an introduction, Mathematical Surveys and Monographs, vol. 110, American Mathematical Society, Providence, RI, 2004.
[3] Bennett Chow, Sun-Chin Chu, David Glickenstein, Christine Guenther, James Isenberg, Tom Ivey, Dan Knopf, Peng Lu, Feng Luo, and Lei Ni, The Ricci flow: techniques and applications. Part I, Mathematical Surveys and Monographs, vol. 135, American Mathematical Society, Providence, RI, 2007. Geometric aspects.
[4] Bennett Chow and Sun-Chin Chu, A geometric interpretation of Hamilton's Harnack inequality for the Ricci flow, Math. Res. Lett. 2 (1995), no. 6, 701–718.
[5] Richard S. Hamilton, Three-manifolds with positive Ricci curvature, J. Differential Geometry 17 (1982), no. 2, 255–306.
[6] Richard S. Hamilton, The formation of singularities in the Ricci flow, Surveys in differential geometry, Vol. II (Cambridge, MA, 1993), 1995, pp. 7–136.
[7] Richard S. Hamilton, Four-manifolds with positive curvature operator, J. Differential Geom. 24 (1986), no. 2, 153–179.
[8] Richard S. Hamilton, The Harnack estimate for the Ricci flow, J. Differential Geom. 37 (1993), no. 1, 225–243.
[9] Richard S. Hamilton, A compactness property for solutions of the Ricci flow, Amer. J. Math. 117 (1995), no. 3, 545–572.
[10] Peter Topping, Lectures on the Ricci flow, London Mathematical Society Lecture Note Series, vol. 325, Cambridge University Press, Cambridge, 2006.
[11] Bruce Kleiner and John Lott, Notes on Perelman's papers, Geom. Topol. 12 (2008), no. 5, 2587–2855.
[12] Huai-Dong Cao and Xi-Ping Zhu, A complete proof of the Poincaré and geometrization conjectures— application of the Hamilton-Perelman theory of the Ricci flow, Asian J. Math. 10 (2006), no. 2, 165–492.
[13] John Morgan and Gang Tian, Ricci flow and the Poincaré conjecture, Clay Mathematics Monographs, vol. 3, American Mathematical Society, Providence, RI; Clay Mathematics Institute, Cambridge, MA, 2007.
[14] Grisha Perelman, The entropy formula for the Ricci flow and its geometric applications, arXiv Mathematics e-prints (November 2002), math/0211159, available at math/0211159.


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| **Date and Place:** | Tue 10:15–11:45, SR 221, Sedanstr. 19 |
| **Problem Classes:** | Tue 12:15–13:45, SR 205, Sedanstr. 19 |
| **Starting on:** | 15 October 2024 |